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மேல் மாகாணக் கல்வித் திணைக்களம்
Department of Education - Western Province

තෙවන වාර පරීක්ෂණය
மூன்றாம் தவணைப் பரீட்சை - **2026**
Third Term Test

ශ්‍රේණිය / தரம் / Grade: 13	විෂයය / பாடம் / Subject: **Combined Mathematics**	පත්‍රය / வினாத்தாள் / Paper: I	පැය / மணித்தியாலம் / Hours: 3

Instructions:

- This question paper consists of two parts;
- **Part A** (Questions **1 -10**) and **Part B** (Questions **11-17**)
- **Part A**
  Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space needed.
- **Part B**
  Answer **five** questions only. Write your answers on the sheets provided.
  At the end of the time allotted, tie the answer scripts of two parts together so that **Part A** is on the top of **Part B** and hand them over to the supervisor.
- You are permitted to remove only **Part B** of the question paper from the Examination Hall.

For Examiners' Use only

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

Total	
In Numbers	
In Words	

Part A

1. Using the Principle of Mathematical induction prove that $\sum_{r=1}^{n}(n+r)=\dfrac{n(3n+1)}{2}$ for $n \in \mathbb{Z}^{+}$

2. Sketch the graphs of $y=3-|x|$ and $y=|x-1|$ in the same diagram. **Hence,** find all the real values of x satisfying the inequality $|x|+|x-2| \leq 6$

3. Sketch the locus C, on the Argand diagram representing the complex numbers such that $\text{Arg}\left(\dfrac{z-1}{z+1}\right)=\dfrac{\pi}{2}$; for $z\neq -1$. **Hence,** find the maximum value of z lie on C such that $|z+2026i|$.

4. Consider the binomial expansion, $\left(\dfrac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\dfrac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$. Show that the term independent of x is 210.

5. Show that $\lim_{x\to 0}\left(\dfrac{(\pi x^2-1)^4-\cos 2x}{\sin^2(\pi x)}\right)=\dfrac{2(1-2\pi)}{\pi^2}$;

6. Show that the region S, enclosed by the curves $y=\dfrac{\cos x-\sin x}{2+\sin 2x}$, $y=0$, $x=0$ and $x=\dfrac{\pi}{4}$, is $\left(a-\dfrac{\pi}{4}\right)$ where a is a constant to be determined.

Hence, deduce the volume of the solid thus generated, when the region enclosed by $y=\sqrt{\dfrac{\cos x-\sin x}{2+\sin 2x}}$, $y=0$, $x=0$ and $x-\dfrac{\pi}{4}$ is rotated about the x - axis by 2π radians.

7. Let the curve given parametrically by, $x=\theta+\cos^2\theta$ and $y=\sin\theta$ for $0\leq\theta<\dfrac{\pi}{2}$. Find $\dfrac{dy}{dx}$ in terms of θ. **Hence,** find the gradient of the tangent which drawn to the curve at $\theta=\dfrac{\pi}{4}$.

8. Let $L_1\equiv 3x+11y-3=0$ and $L_2\equiv 12x+5y+1=0$. It is given that L_1, L_2, and L_3 are concurrent at the point A and L_2 and L_3 make equal angle with the line L_1. Without finding the coordinates of A explicitly find the equation of L_3.

9. A circle with centre lie on x - axis and radius 5 units passes through the point (4, 5) Show that two such Circles can exist. Find both equations of them. Further show that the area of the triangle, which make common chord with axes is $\frac{81}{2}$ square units.

10. Let $T(x) \equiv \cos x(\cos x + \sqrt{3}\sin x)$ for all $x \in \mathbb{R}$. Find the values of a, b and $\alpha \left(0 < \alpha < \frac{\pi}{2}\right)$ such that $T(x) \equiv a + b.\cos(2x - \alpha)$. **Hence** plot the graph of $y = T(x)$ in $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right)$.

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Part B

- Answer **five** questions only.

11. (a) Given that the roots of the quadratic equation $ax^2+bx+c=0$ are real, where $a, b, c \in \mathbb{R},$ and $a \neq 0$ without solving the equation, show that $\Delta \geq 0$

Let $f(x)=(x^2+4x-5)+k(1+(k-1)x-kx^2)$ for $k \in \mathbb{R}-\{-1,1\}$ Show that the roots of the quadratic equation $f(x)=0$, are real. Show that the roots of the quadratic equation $f(x)=0$ can never both be negative for $k \neq 0$.

Further, when $k \in \mathbb{Q}$, find the value of k such that the difference between the roots of $f(x)=0$ is 1.

(b) Let f is a polynomial function of degree 3. Given that $3x+2$ is a factor of $f(x)$ and $3x+2$ is remainder when $f(x)$ divided by x^2+x+1

Find $f(x)$ such that the image of 1 under the function f is 10. Further, Solve the equation $f(x-1)=-x$

12. (a) A vendor has two bags, each containing three mangoes and three pineapples. He is required to select six fruits from these bags in order to prepare a fruit basket. The fruit basket should include no more than two Pineapples.

(i) Find the number of different fruit baskets that can be formed if an even number of fruits must be selected from each bag.

(ii) Find the number of different fruit baskets that can be formed if exactly one pineapple must be selected for the fruit basket.

(b) Determine the values of integers A and B such that, $A(x^2+x+1)+B(x^2-x+1)\equiv x^2+3x+1$ for $x \in \mathbb{R}$. Write down r^{th} term U_r ; $r \in \mathbb{Z}^+$ of the series

$$\frac{5}{1^4+1^2+1}\left(\frac{1}{2}\right)+\frac{11}{2^4+2^2+1}\left(\frac{1}{4}\right)+\frac{19}{3^4+3^2+1}\left(\frac{1}{8}\right)+\cdots$$

Write down $f(r)$, such that $U_r=f(r-1)-f(r)$ and show that

$$\sum_{r=1}^{n}U_r=1-\frac{1}{(n^2+n+1).2^n}; \quad \text{for } n \in \mathbb{Z}^+$$

Show that the infinite series $\sum_{r=1}^{\infty}U_r$ is convergent and find its sum.

Hence deduce the value of $\sum_{r=1}^{\infty}\left(\frac{2^r.r^4+(2^r+1).r^2+2^r+3r+1}{2^r.(r^4+r^2+1)}\right)$

13. (a) Let $A=\begin{pmatrix} a & 1 & 3 \\ -1 & -1 & a \end{pmatrix}$, $B=\begin{pmatrix} 1 & -1 & b \\ b & -1 & 0 \end{pmatrix}$ and $C=\begin{pmatrix} 4 & 1 \\ 2 & 0 \end{pmatrix}$ where $a,b \in \mathbb{Z}$ Determine a and b such that $AB^T = C$ for these values of a and b, show that $B^T A=\begin{pmatrix} 1 & 0 & 5 \\ -1 & 0 & -5 \\ 2 & 1 & 3 \end{pmatrix}$

Further, find the matrix D such that $A^2+2I=CD$ where I is the identity matrix of order 2 × 2 and B^T is the transpose of B.

(b) In an Argand diagram, the point A and B represent the complex numbers 3 + i and 7 + 4i respectively. It is given that ABCD is a rhombus such that point A lies on the real axis and point D lies on the imaginary axis. If it is given that $A\hat{D}C = \tan^{-1}\left(\frac{7}{24}\right)$, find the complex number represented by the points C and D.

(c) Let $z=\frac{1}{2}(3-\sqrt{3}i)$. Express z in the form $z=r(\cos\theta+i\sin\theta)$ where $r\,(r>0)$ and $\theta\left(\frac{-\pi}{2}<\theta<\frac{\pi}{2}\right)$ to be determind.

Using De Moivre's theorem or otherwise determine n such that $z^n+\overline{z}^n=1458$ for $n \in \mathbb{Z}^+$

14. (a) Let $f(x)=\frac{ax+b}{(x+a)^2}$; for $x \neq -a$ and $(a, b) \in \mathbb{Z}^+$

Show that for any value of a, the graph of a $y = f(x)$ has a horizontal asymptote at $y = 0$

If it is given that the graph of $y=f(x)$ has a vertical asymptote at $x = -2$, find the value of a. If it is given that the turning point of he graph of $y=f(x)$ is $A \equiv (-1, a-1)$ show that $b = 3$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\frac{-2(x+1)}{(x+a)^3}$.

Hence, write down the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

It is given that $f''(x)=\frac{2(2x+1)}{(x+a)^4}$

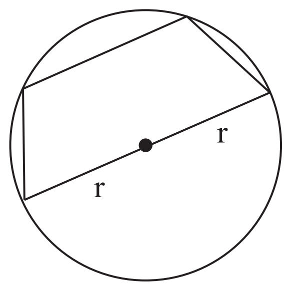
Hence write down the intervals on which $f(x)$ is concave up and concave down.

Find the coordinates of inflection point of $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the turning point, the asymptotes and the point of inflection.

(b) As shown in the adjoining figure,

a trapezium forming a cyclic quadrilateral is inscribed in a circle of radius r such that one of its parallel sides Coincides with a diameter of the circle. Find the dimensions of the trapezium for which its area is maximum.



15. (a) Find the values of the constants A, B and C such that,

$Ax(3x^2+4)-(Bx+C)(3x-1)^2 \equiv 15x^2+5x+1$ for all $x \in \mathbb{R}$.

Let $\mathrm{I}(x) = \dfrac{15x^2+5x+1}{(3x-1)^2(3x^2+4)}$ for $x \neq \dfrac{1}{3}$

Express $I(x)$ in partial fraction and find $\int \mathrm{I}(x).dx$

(b) Let $I_n = \int x^n.(Inx)^2\ .dx$ for $n \in \mathbb{Q}$. Using integration by parts repeatedly, show that,

$I_n = x^{n+1}\left(\dfrac{[f(x)]^k}{\eta+1} + \dfrac{[f(x)]^{k-1}}{(\eta+1)^2} + \dfrac{2[f(x)]^{k-2}}{\eta(\eta+1)3}\right) + C$ for $n \neq -1$ where k is a positive integer to be determine, $f(x)$ is a function of x to be determine and C is a given real constant.

Hence, deduce that $\int_1^e \sqrt{x}\,(\ln x)^2.dx = \dfrac{2}{27}(9\sqrt{e^3}-8)$

(c) Let $I = \displaystyle\int_0^{\pi} \frac{x.\sin x}{1+\cos^2 x}.dx$

Using the relation $I = \displaystyle\int_0^a f(x).dx = \int_0^a f(a+b-x).dx$ for $a>0$ or otherwise show that

$I = \dfrac{\pi}{2}\displaystyle\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}.dx$

Hence, deduce that, $\displaystyle\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\ .dx = \frac{\pi^2}{4}$;

16. Let $A \equiv \left(\dfrac{\sqrt{2}+1}{\sqrt{2}}, \dfrac{\sqrt{2}+1}{\sqrt{2}}\right)$ and L be the straight line passing through the point A, which make an intercepts of $2+\sqrt{2}$ on the coordinate axes.

Show that the circle S, of smallest, radius which touches the coordinate axes and the straight line L is, $S \equiv x^2+y^2+\lambda x+\lambda y+1=0$ where λ is an integer to be determine.

If B be the point on the circle S, nearest to the origin O, find the Coordinates of B.

Write down the equation of the straight line passing through B, which bisect the radii drawn from center of S to coordinate axes.

Show that these straight lines make a angle of $\tan^{-1}(2\sqrt{2}-1)$ with line L = 0

Furthermore, find the equation of the circle which intersect the circle. S = 0 orthogonal and whose centre lie on x axis

17. (a) Write down $\sin 3\theta$ in terms of $\sin\theta$. Hence obtaing $\cos 3\theta \equiv 4\cos^3\theta - 3\cos^3\theta$

Further, deduce that, $\sin 20^0.\sin 40^0.\sin 60^0.\sin 80^0 + \cos 20^0.\cos 40^0.\cos 60^0.\cos 80^0 = \frac{1}{4}$

(b) Given that in the usual natation for a triangle ABC, $\cos A.\cos B + \sin A.\sin B.\sin C = 1$.

Show that $\dfrac{1-\cos A.\cos B}{\sin A.\sin B} \leq 1$.

Hence, deduce A = B

Further, find the value of *k* such that a : b : c = 1 : 1 : *k* where *k* is real irrational constant.

(c) (i) Show that, $\tan^{-1}(A) - \tan^{-1}(B) \equiv \tan^{-1}\left(\dfrac{A-B}{1+AB}\right)$;

(ii) Show that, $\dfrac{3\sin 2\theta}{5+4\cos 2\theta} \equiv \dfrac{2\left(\frac{\tan\theta}{3}\right)}{1+\left(\frac{\tan\theta}{3}\right)^2}$;

Using the results of part (i) and (ii) or otherwise, solve the equations

$\theta = \tan^{-1}(2\tan^2\theta) - \dfrac{1}{2}\sin^{-1}\left(\dfrac{3\sin 2\theta}{5+4\cos 2\theta}\right)$;