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**Education Department – Western Province**

තෙවන වාර පරීක්ෂණය - 13 ශ්‍රේණිය, 2026 ජූලි

**Third Term Test – Grade 13, 2026 July**

සංයුක්ත ගණිතය II

**Combined Mathematics II**

**10** **E** **II**

පැය තුනයි

***Three Hours***

අමතර කියවීම් කාලය - මිනිත්තු 10 යි

**Additional Reading Time - 10 minutes**

**Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will priorities.**

**Name:**........................................................................................... **Class :**..........

**Instructions:**

- *This question paper consists of two parts;*
  Part **A** (Questions **1 -10**) and **Part B** (Questions **11-17**)
- **Part A**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space needed.*
- **Part B**
  *Answer **five** questions only. Write your answers on the sheets provided.*
- *At the end of the time allotted, tie the answer scripts of two parts together so that* **Part A** *is on the top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove **only*** **Part B** *of the question paper from the Examination Hall.*

**For Examiners' Use only**

| (10) Combined Mathematics II | | |
|---|---|---|
| **Part** | **Question No.** | **Marks** |
| A | 1 | |
| | 2 | |
| | 3 | |
| | 4 | |
| | 5 | |
| | 6 | |
| | 7 | |
| | 8 | |
| | 9 | |
| | 10 | |
| B | 11 | |
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| | 16 | |
| | 17 | |
| | **Total** | |

**Total**

| | |
|---|---|
| **In Numbers** | |
| **In Words** | |

[*see page two*

**Part A**

1. Two particles $P$ and $Q$, each of mass $m$, move towards each other along the a straight line on a smooth horizontal floor and collide directly. Just before the collision, the velocities of $P$ and $Q$ are $u$ and $nu$ respectively. After the collision, the direction of motion of particle $Q$ is reversed, and due to colliding with a wall located at some distance from the particle, the velocity of $Q$ becomes half of its initial velocity. If $n = \frac{1}{2}$, find the coefficient of restitution between the wall and particle Q. The coefficient of restitution between the two particles is $\frac{1}{5}$.

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2. An object gently released from an aircraft moving at a uniform speed $v$ at a height $H$ from the ground level, just clears the top of a building of height $h$. Show that the building is located at a horizontal distance of $v\sqrt{\frac{2(H-h)}{g}}$ from the position O where the object was released.

   Furthermore, if the object hits the ground at a distance twice the above horizontal distance from $O$, find the time of flight of the object

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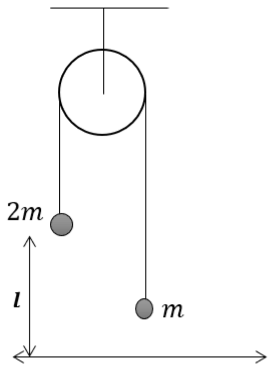
[*see page three*

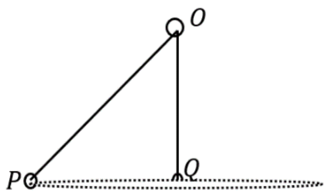
3. Two particles $A$ and $B$ of masses $2m$ and $m$ respectively, are attached to the two ends of a light inextensible string passing over a fixed smooth pulley. As shown in the figure, particle A is kept in equilibrium at a height $l$ from a horizontal floor and released from rest. Show that the system then moves with a uniform acceleration of $\frac{g}{3}$ and particle A collides with the floor. If the coefficient of restitution between particle A and the floor is $\frac{1}{2}$, show that the impulsive reaction created on the string due to the collision is $m\sqrt{6gl}$.




4. A motor car of mass $M$ travels up an inclined road at a maximum speed of $v$. The inclination of the road to the horizontal is $\sin^{-1}\left(\frac{1}{g}\right)$. The resistance to motion is $R$. Find the power of the car. Now, a trailer of mass $m$ is attached to this motor car by a light tow-bar. The motor car moves with a constant acceleration $a$ along a horizontal road while working at the same power as above. The resistance to the car remains unchanged. Find the tension in the tow-bar when the speed of the car is $2v$.

[*see page four*

5. Two particles $P$ and $Q$ of masses $m$ and $2m$ respectively, are connected to the two ends of a light inextensible string of length $3l$ which passes through a smooth fixed ring $O$. As shown in the figure, if the particle $P$ describes a horizontal circle with a constant angular velocity ω about $Q$ as the center, find the inclination of the string portion $OP$ to the vertical. Furthermore, show that $\omega = \sqrt{\frac{g}{2l}}$ .

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6. In usual notation, the position vectors of two points $A$ and $B$ with respect to the origin $O$ are $\lambda\boldsymbol{i} + \mu\boldsymbol{j}$ and $5(\boldsymbol{i} + \boldsymbol{j})$ respectively, where $\lambda, \mu \in \mathbb{R}^+$. A point $C$ lies on the line segment AB such that $AC: CB = 3: 1$. If $\overrightarrow{OC} = \frac{9}{2}\boldsymbol{i} - 3\boldsymbol{j}$, find the values of $\lambda$ and $\mu$.

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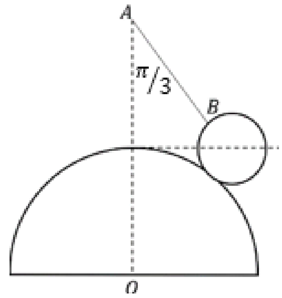
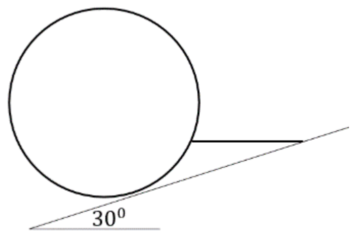
[*see page five*

7. A smooth hemisphere of radius $2a$ is fixed with its rim horizontal and its vertex uppermost. A smooth sphere of radius $a$ and weight $W$ is kept in equilibrium on its convex surface by a light inextensible string $AB$ tied to a point $A$ vertically above the center $O$ of the hemisphere, and to a point $B$ on the sphere. In the equilibrium position, the center of the sphere and the vertex of the hemisphere lie on the same horizontal line. Find the tension in the string and the reaction between the sphere and the hemisphere.




8. A rough sphere of radius $a$ and weight $W$ is placed on a rough plane inclined at an angle of $\frac{\pi}{6}$ to the horizontal. It is held by a light elastic string of natural length $l$ and modulus of elasticity $W$, attached to a point on the plane. If the coefficient of friction between the plane and the sphere is $\frac{1}{4}$, find the extension of the elastic string when the sphere is in limiting equilibrium with the string remaining horizontal.




[see page six

9. Let $A$ and $B$ be two events in a sample space $\Omega$. It is given that $P(A) = 2p, P(B) = p$ and $P(A \cup B) = \frac{5P}{2}$, where $p > 0$. Find $P(A \cap B)$ in terms of $p$. If the events $A$ and $B$ are independent, obtain the value of $p$.

10. The mean of three distinct integers, each less than $10$, is $4$. When the three numbers are arranged in ascending order, the first two numbers are consecutive. If a new integer, equal to twice the middle term, is introduced, the mean of the four numbers becomes $5$. Find the four integers.

[*see page seven*

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**Education Department – Western Province**

තෙවන වාර පරීක්ෂණය - 13 ශ්‍රේණිය, 2026 ජූලි

**Third Term Test – Grade 13, 2026 July**

සංයුක්ත ගණිතය II

**Combined Mathematics II**

**10** **E** **II**

**Part B**

❖ Answer **Five** questions only.

11. **(a)** A particle $P$ is projected vertically upwards from a point $O$ with an initial velocity $u$. At the same instant, another particle $Q$ is gently released from rest from a point situated at a height $h$ vertically above $O$. Sketch the velocity-time graphs for the motions of $P$ and $Q$ on the same diagram. **Hence,** show that:

I. $P$ and $Q$ meet each other after a time $t_1 = h/u$.

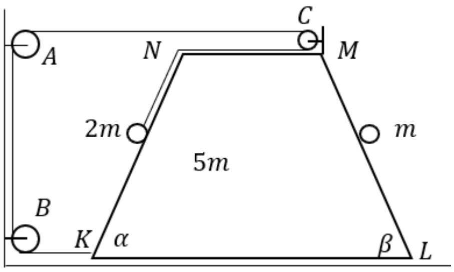
II. At this instant, the displacement of $P$ is $\left(2u^2 - gh\right) h/2u^2$ .

After a time $T$, such that $h/u < T < u^2/2g$, another particle $R$ is projected vertically upwards from $O$ with a velocity $v$. Draw the velocity-time graph for the motion of $R$ on the same diagram. If $Q$ and $R$ meet each other after a time $t_1$ from the start, show that $v = u - gT - \frac{gT^2u}{2h}$. At this instant, at what height is $P$ located?

**(b)** In a still sea, a ship $P$ travels due East with a uniform speed $v$, while a ship $Q$ travels due North with a uniform speed $2u$. At a certain instant, ship $Q$ is located at a distance of $d\ km$ in a direction South-East from $P$. At the same instant, another boat $R$ is located at a distance of $\frac{d}{\sqrt{2}}\ km$ due South from $P$, and boat $R$ moves along a straight path with a uniform speed of $3u$ to intercept ship Q. Find the time taken for $R$ and $Q$ to meet.

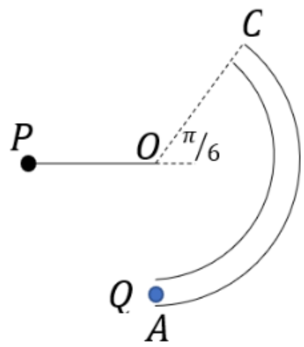
If $v < \sqrt{2}u$, show that ships $P$ and $Q$ will never meet at any instant. Show also that the shortest distance between $P$ and $Q$ is given by $d \sin\left(\frac{\pi}{4} - \theta\right)$, where $\theta = \tan^{-1}\left(\frac{v}{2\sqrt{2}u - v}\right)$.

12. **(a)** In the figure, $KLMN$ is a cross-section through the centre of gravity of a smooth uniform wedge of mass $5m$, which is placed with its face $KL$ on a smooth horizontal floor. $N\hat{K}L = \alpha$ and $K\hat{L}M = \beta$. The lines $NK$ and $LM$ are the lines of greatest slope of their respective faces. $A$ and $B$ are fixed pulleys attached to a vertical wall, and $C$ is a light pulley attached to the wedge at $M$. As shown in the diagram, a light inextensible string passing over the pulleys has one end attached to point $K$ and the other end attached to a particle of mass $2m$ lying on $NK$. Another particle of mass $m$ is placed on $ML$, and the system is released from rest. Obtain sufficient equations to determine the tension in the string, the acceleration of the particles, and the acceleration of the wedge.




[see page eight

**(b)** One end of a light inextensible string of length $l$ is tied to a fixed-point O, and a particle $P$ of mass $m$ is attached to its other end. The string is held taut, horizontally and particle P is projected vertically downwards with a velocity $u$.




When the string becomes vertical, it strikes a particle $Q$ of mass $2m$ placed at the lower end $A$ of a smooth tube of radius $l$.

If the coefficient of restitution between the particles is $\frac{1}{2}$, show that particle P comes to rest immediately after the impact. In the subsequent motion inside the tube, find the velocity of particle Q and the normal reaction between the tube and Q when the radius passing through Q makes an angle θ with the downward vertical. Show also that if particle Q just reaches C, then $u = \sqrt{10gl}$.

13. **(a)** One end of a light elastic string of natural length $2a$ and modulus of elasticity $8mg$ is attached to a fixed point $O$, which is at a height of $3a$ vertically above a horizontal floor. The other end of the string is attached to a particle P of mass $m$. $D$ is a point vertically below $O$ such that $OD = 2a$. Find the extension of the string when the particle hangs freely in equilibrium. Now, the particle is held at O and projected vertically downwards with a velocity of $\sqrt{5ga}$. Find the velocity of $P$ at $D$. Show that the equation of motion of the particle P is given by $\ddot{x} = -\omega^2\left(x - \frac{9a}{4}\right)$, where $OP = x$, $(2a < x < 3a)$ and $\omega(> 0)$ is a constant to be determined. By taking $X = x - \frac{9a}{4}$, show that $\ddot{X} + \omega^2 X = 0$, and express the frequency of this simple harmonic motion.

Using the formula $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude $c$ of this motion, the velocity of $P$ as it reaches the floor, and the time taken to travel from $D$ to the floor.

The particle $P$ collides inelastically with the floor, and at that instant, it coalesces with a particle $Q$ of equal mass $m$ lying gently on the floor. A second light elastic string of natural length $\frac{a}{2}$ and modulus of elasticity $8mg$ has one end attached to $Q$ and the other end attached to a point $A$ on the floor vertically below $O$.

It is given that when the composite particle PQ moves upwards from the floor, its equation of motion for $\frac{5a}{2} < w < 3a$ is given by $\ddot{w} = -\frac{2g}{a}\left(w - \frac{5a}{2}\right)$. State the centre of oscillation and the amplitude of this motion, and find the velocity of the composite particle at point $B$, which is at a distance $\frac{5a}{2}$ vertically below O.

Show that when the composite particle moves upwards from $B$, its equation of motion is given by $\ddot{y} + \frac{10g}{a} y = 0$, where $y$ is the displacement measured upwards from $B$. Show further that in the subsequent motion, the composite particle cannot reach to $D$.

[*see page nine*

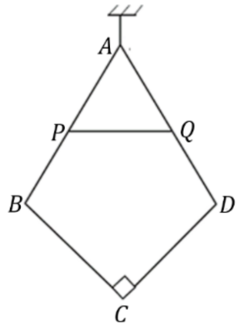
14. **(a)** The position vectors of three points $A, B$ and $C$ with respect to the origin $O$ are $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c} = \lambda \boldsymbol{a} + \mu \boldsymbol{b}$ respectively, where $\lambda, \mu \in \mathbb{Z}^+$and $|\overrightarrow{OC}| = 2\sqrt{2}$ . It is given that $|a| = |b| = \sqrt{\frac{2}{3}}$, $A\hat{O}B = \frac{\pi}{3}$,, and $OC$ is perpendicular to $AB$. The point $P$ divides the line segment $AB$ internally in the ratio $2:3$, and the point $Q$ divides the line segment $OC$ externally in the ratio $3:1$. Find the position vectors of the points $P$ and $Q$ in terms of $a$ and $b$.

Show that $\mu|\boldsymbol{b}|^2 - \lambda|\boldsymbol{a}|^2 + (\lambda - \mu)\, \boldsymbol{a} \cdot \boldsymbol{b} = 0$.

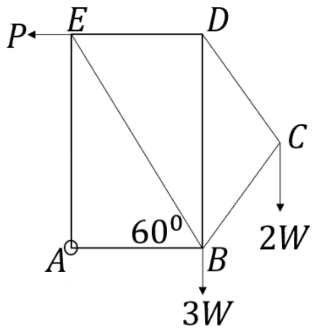
Show that $\lambda = 2$ and find the value of $\mu$. Hence, show that $O, P$ and $Q$ are non-collinear.

**(b)** Forces of magnitudes $6, 3, 6, 3, 17\sqrt{2}$ and $\sqrt{2}\, N$ act along the sides $\overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}$, $\overrightarrow{BA}$, $\overrightarrow{AC}$ and$\overrightarrow{DB}$ respectively of a square $ABCD$ of side length $a\, m$. When a couple of magnitude $4a\, Nm$ acting in the sense $DCBA$ is added to this plane, find the magnitude of the resultant of the system of forces and the angle it makes with $AB$. If the line of action of the resultant intersects $AB$ at $N$, find the length $AN$.If this system of forces is equivalent to two parallel forces $P$ and $Q$ acting at $A$ and $B$ respectively, find $P$ and $Q$.

15. a) Four uniform rods $AB, BC, CD$ and $DA$, whose masses are proportional to their lengths and have a weight $w$ per unit length, are smoothly jointed at their ends to form a quadrilateral framework. The framework is freely suspended in equilibrium from the vertex $A$ as shown in the diagram. It is given that $AB = AD = 2a$, $BC = CD$, and $B\hat{C}D = \frac{\pi}{2}$. The midpoints of $AB$ and $AD$ are connected by a light rod $PQ$ of length $a$. Find the reaction at the joint $C$. Also, find the tension in the light rod.




**(b)** The framework shown in the adjacent diagram consists of light rods $AB, BC, CD, DE, AE, BE$ and $BD$ freely jointed at $A, B, C, D$ and $E$. It is given that $AB = BC = CD = DE = 2a$,$AE = BD$, and $A\hat{B}E = \frac{\pi}{3}$. Weights of $3W$ and $2W$ are suspended at $B$ and $C$ respectively. The framework is smoothly hinged to a fixed point at $A$ and is maintained in equilibrium with $AB$ and $ED$ horizontal by a horizontal force $P$ applied at $E$. Find the value of $P$ in terms of $W$. Using Bow's notation, draw a stress diagram. Hence, determine the stresses in the rods $BC, BD, CD, DE$ and $BE$, stating whether they are tensions or thrusts.

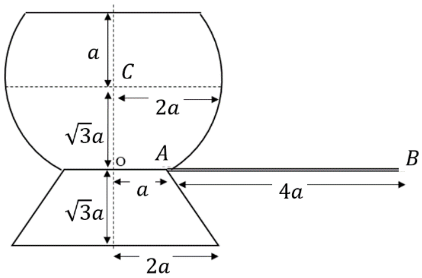



16. **Using integration,** show that

(i) The mass of a uniform solid right circular frustum of a cone of height $h$, base radius $2a$, top radius $a$, and density $\rho$ is $\frac{7\pi a^2 h}{3}\rho$, and its centre of mass lies at a distance of $\frac{11}{28}h$ from the centre of its base.

(ii) the mass of frustum made from a uniform hollow hemispherical shell of radius $2a$ and surface density ρ, by cutting parallel to the plane of its circular rim, perpendicular to its axis, and at a distance of $2a\cos\alpha$ from its centre $O$ is $8\pi a^2 \rho \cos\alpha$, and its centre of gravity lies on its axis at a distance of $a\cos\alpha$ from O.

An object S is constructed from a uniform hollow sphere of radius $2a$, center $C$, and surface density $a\rho$ by removing two portions cut by planes perpendicular to its vertical diameter, located at a height $a$ above $C$ and at a depth $\sqrt{3}a$ below $C$ respectively.




A uniform solid right circular frustum of a cone of base radius $2a$, top radius $a$, height $\sqrt{3}a$, and density $\rho$ is attached to the lower opening of $S$ as shown in the diagram. A uniform horizontal rod AB of length $4a$ and linear density $\pi a^2\rho$ is fixed at the point $A$. Show that the centre of mass of the composite body lies at a distance of $\frac{3\left(16\sqrt{3}+15\right)}{4k}a$ from $OA$ and at a distance of $\frac{36}{k}a$ from $OC$, where $k = 24 + 19\sqrt{3}$. Furthermore, show that when the object is freely suspended from the point $B$, the inclination of $AB$ to the vertical is given by: $\tan^{-1}\left(\frac{3\left(15+16\sqrt{3}\right)}{4\left(5k-36\right)}\right)$.

**17. (a)** In a refrigerator manufacturing plant, refrigerators are produced exclusively by three machines, A, B, and C. Machine A produces 50% and machine B produces 30% of the total production. It is given that the percentages of defective refrigerators produced by machines A, B, and C are 2%, 4%, and 5% respectively. Find the probability that a refrigerator selected at random:

I. is defective.
II. was produced by machine B, given that it is found to be defective.

A new testing device is introduced to the factory. The probability that it correctly identifies a defective refrigerator is 0.95. However, the probability that it incorrectly classifies a non-defective refrigerator as defective is 0.03. A randomly selected refrigerator is tested using this new device. Find the probability that the refrigerator is truly defective, given that it is identified as defective by the device.

**(b)** The table below shows the number of units produced by 200 employees of the same factory:

| No. of Units | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 |
|---|---|---|---|---|---|
| No. of Employers | 20 | 50 | 80 | 30 | 20 |

For the distribution given in this table, find the mode $M_0$ and the median $M_a$.

By taking the assumed mean as $A = 50$, estimate the mean $\mu$ and the standard deviation $\sigma$ of the distribution. Furthermore, estimate the coefficient of skewness $k$, defined by $k = \frac{\mu - M_0}{\sigma}$.

All the employees who were in the 80–100 range were assigned to supervisory duties in addition to unit production, the number of units produced by them fell into the 0–20 range. Find the mean of the new distribution.

Without calculating the new standard deviation, state with reasons whether the new coefficient of skewness is greater than or less than $k$. Justify your answer.